Name:____

This assignment is worth 100 points. You will be awarded 40 points for attempting the entire assignment (that is answer all problems). All problems will be graded for the remaining 60 points. The space left between each question is indicative of how much work you should show. If there are any problems you find particularly difficult, circle them in red. If there are any particular questions you would like feedback on, circle them in green. These are examples of questions that might appear on an exam/quiz. If you use a calculator to help, make sure you can also do them without it.

Seeing as everybody decided to go home the class before fall break, this homework will cover everything that was planned for that day, as well as tying it in with the logarithm stuff we have seen in class.

Compound Interest

Money deposited in a savings account increases exponentially, because the interest on the account is calculated by multiplying the amount in the account by a fixed factor - the interest rate. *Compound interest* means that the interest earned in one time period is split into smaller increments.

If an amount P is invested at an annual interest rate r compounded n times each year, then the amount $A(t)$ of the investment after t years is given by the formula

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}.$$

The ***Annual Percentage Yield*** is the growth rate of the above exponential model, $\frac{r}{n}$.

What does that mean?

- Amount P: initial amount invested
- Interest rate r: interest earned on investment
- Compounded n times: how many installments the interest is paid in per time period
- Amount $A(t)$: how much is currently in the account

Example:

Let's suppose that \$1000 is deposited into a savings account paying 6% interest annually, compounded monthly. We want to find a model $A(t)$ that represents the amount in the account after t years and the annual percentage yield.

- Initial value: \$1000
- Interest rate: 6%
- Compounded: Monthly
- Time period: Year = 12 Months

The key thing to note is how your installments relate to your time period. In this example there are 12 installments, because there are 12 months in 1 year. So, the model we build is

$$A(t) = 1000\left(1 + \frac{0.06}{12}\right)^{12t} = 1000(1.005)^{12t}.$$

The annual percentage yield is simply the growth rate; $\frac{0.06}{12} = 0.005$.

The model that we have built will tell us the amount of the investment after t years. So, after 3 years, the value of the above investment would be

$$A(3) = 1000(1.005)^{12(3)} = 1000(1.005)^{36} = 1000(1.19668052) = \$1196.68.$$

1. Look at problems $13 - 20$ on worksheet 7. Try them. Ask questions on Monday.
2. Pete is deciding which account to invest his \$3000 in. Below are the details of each account.
   - Account A offers a 4% annual interest rate, compounded monthly
   - Account B offers a 4% annual interest rate, compounded quarterly (1 quarter = 3 months)

   (a) If Pete plans on leaving his money in the account for 3 years, which account would you advise he goes with?

   Answer:____________________

   (b) What is the annual percentage yield of each account?

   Answer:____________________

   Answer:____________________

Continuously Compounded Interest

From class we have seen the number $e \approx 2.7183$. An application of where this number can arise is Continuously Compounded Interest. We will see that it is very similar to Compound Interest.

> If an amount P is invested at an annual interest rate r compounded continuously, then the amount $A(t)$ of the investment after t years is given by the formula
>
> $$A(t) = Pe^{rt}.$$

Example:

As before, let's suppose that \$1000 is deposited into a savings account paying 6% interest annually, but this time we will create a model when interest is compounded continuously. Well, this model would be given by

$$A(t) = Pe^{rt} = 1000e^{0.06t}.$$

You can check, if you wish, that if you increase the number of installments in a compound interest model, you will get closer and closer to the value of the continuously compounded model.

3. If \$25,000 is invested at an interest rate of 7% per year, find the models that represent the amount of the investment $A(t)$ for the following compounding methods;

   (a) Semiannually (twice per year):

   Answer:____________________

   (b) Quarterly:

   Answer:____________________

   (c) Monthly:

   Answer:____________________

   (d) Continuously

   Answer:____________________

4. If \$20,000 is invested at an interest rate of 5% per year, find the amount of the investment after 4 years for the following compounding methods;

   (a) Semiannually:

   Answer:____________________

   (b) Quarterly:

   Answer:____________________

   (c) Monthly:

   Answer:____________________

   (d) Continuously

   Answer:____________________

The rest of this assignment is about exponential models in terms of e.

The exponential growth or decay model

$$f(x) = Ca^x \text{ (growth or decay factor } a \text{ per time period)}$$

is equivalent to the exponential model

$$f(x) = Ce^{rt} \text{ (instantaneous growth or decay rate } r \text{ per time period)}$$

where the instantaneous growth or decay rate is

$$r = \ln(a).$$

Example:

Express the model $f(x) = 350(1.4)^x$ in terms of the base e. For this model, the growth factor is $a = 1.4$, thus the instantaneous growth rate is

$$r = \ln(a) = \ln(1.4)$$

So the model we are after is

$$f(x) = Pe^{rx} = 350e^{\ln(1.4)x}.$$

5. Food poisoning is often caused by E. coli bacteria. To test for the presence of E. coli in a pot of beef stew, a biologist performs a bacteria count on a small sample of the stew kept at 25°C. She determines the count is 5 units per millilitre and the number will double every 40 minutes.

   (a) Find the *hourly* growth factor a and find an exponential model $f(t) = Ca^t$ for the bacteria count in the beef stew.

   Answer:____________________________

   (b) Find the instantaneous growth rate r and find an exponential model $g(t) = Ce^{rt}$ for the bacteria count t hours later.

   Answer:____________________________

   (c) Using you model from part b), determine how many hours it would take for the amount of bacteria to reach 25 units.

   Answer:____________________________

6. Internet World Stats reports that the number of internet users in China increased by 1024% from 2000 to 2008 (so the 8 year growth rate is 10.24 and the 8 year growth factor is 11.24). The number of internet users in 2000 was 22.5 million. Assume that the number of internet users increases exponentially.

   (a) Find the *yearly* growth factor a and find an exponential model $f(t) = Ca^t$ for the number of internet users in China after t years since 2000, where the number of users is measured in millions.

   Answer:______________________________

   (b) Find the instantaneous growth rate r and find an exponential model $g(t) = Ce^{rt}$ for the number of internet users in China after t years since 2000.

   Answer:______________________________

   (c) Using your model from part b), what year would you expect the number of users in China to reach 450 million?

   Answer:______________________________

The End.